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Students' Mathematical Argumentation with the Visualizer Cognitive Style in Proving the Congruence of Triangles

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Abstract: The use of axiomatic deductiveism, perception problems, mathematical visual representations and how students convey arguments as a bridge in proving geometric problems are obstacles that are often faced by students. This research aims to describe how prospective mathematics teacher students who have a visualiser cognitive style convey their mathematical arguments in explaining proofs related to triangle congruence. The subjects of this research were selected from students who have a visualizer cognitive style who had previously been given a cognitive style questionnaire. The research subjects consisted of 3 subjects, each with high (S1), medium (S2) and low (S3) mathematics abilities. Data were analyzed using Toulmin argumentation components. The research results show that S1 provides proof based on correct claims, providing fairly complete data both from the question statement and additional data resulting from the subject's reasoning which is used to decide on the claim. The subject uses several warrants and is supported by logical backing to connect the data with the claim. For S2 and S3 to provide final answers or false claims, they have incomplete data as a basis for deciding claims because the subject is mistaken in representing the image and is deep in mathematical literacy related to errors in describing the visualization in the image.

Keywords: Mathematical Argumentation; Visualizer; Proof; Congruence of Triangles

Abstrak: Penggunaan deduktif aksiomatik, masalah persepsi, representasi visual matematis serta bagaimana mahasiswa menyampaikan argumentasi sebagai jembatan dalam membuktikan masalah geometri menjadi kendala yang sering dihadapi oleh mahasisiwa. Penelitian ini bertujuan untuk mendeskripsikan bagaimana mahasiswa calon guru matematika yang memiliki gaya kognitif visualiser dalam menyampaikan argumentasi matematis mereka dalam menjelaskan pembuktian terkait kongruensi segitiga. Subjek penelitian ini dipilih dari mahasiswa yang memiliki gaya kognitif visualiser yang sebelumnya sudah diberikan angket gaya kognitif. Subjek penelitian terdiri 3 subjek dengan masing -masing berkemampuan matematika tinggi (S1), sedang (S2), dan rendah (S3). Data dianalisis menggunakan komponen argumentasi Toulmin. Hasil penelitian menunjukkan bahwa S1 memberikan pembuktian berdasarkan klaim yang benar, memberikan data yang cukup lengkap baik dari pernyataan soal ataupun data tambahan hasil penalaran subjek yang digunakan untuk memutuskan klaim. Subjek menggunakan beberapa warrant serta didukung oleh backing yang logis untuk menghubungkan data dengan klaim. Untk S2 dan S3 memberikan jawaban akhir atau klaim salah, memiliki data yang kurang lengkap sebagai dasar memutuskan klaim yang disebabkan subjek keliru dalam merepresentasi gambar serta terkendalam dalam literasi matematis terkait dengan kekeliruan dalam membahasakan visualisasi yang ada pada gambar.

Kata Kunci: Argumentasi Matematis; Visualiser; Pembuktian; Kongruensi Segitiga

INTRODUCTION

Geometry as a branch of mathematics is still often a boring topic for some students. This is no exception at the tertiary level. Even though it is actually interesting to study, there are still many students who categorize geometry as a difficult subject (Rezki & Nurfadilah, 2018). Many studies reveal the difficulties and challenges faced by students in studying and solving geometric problems. Some of these include difficulties in drawing or interpreting geometric images and applying more than one theorem to solve certain geometric problems (Casanova & Lapinid, 2021; Dinata, 2019; Dirgantoro, 2019; Juman et al., 2022 ; Ramdjid et al., 2022)

Regarding the difficulty of geometry material, it cannot be separated from the discussion regarding reasoning and proof, which is a challenge that is not easy. Researchers also agree that proof and reasoning are very important in supporting students' mathematical abilities and are important topics in the mathematics curriculum (Beni, 2015; Winer & Battista, 2022). Difficulty in translating mathematical language into geometric models is a problem that is often faced. This is inseparable from difficulties in using concepts, principles, and difficulties in solving verbal problems and related evidentiary problems (Fauzi & Arisetiawan, 2020). Furthermore, the discussion regarding the difficulties in proving geometry cannot be separated from the use of axiomatic deductiveism which is still at the top, then problems of perception, misunderstandings about visual processes and activities, and finally problems of using procedures, concepts and principles (Budiarto & Artiono, 2019).

In compiling mathematical proof, students cannot be separated from the use of argumentation. Argumentation plays an important role in solving mathematical problems. Students must use valid and systematic arguments in compiling a geometric proof (Pramesti & Rosyidi, 2020). Argumentation is an element that has a structure and is a strategy for solving a problem using argumentation model statements (Widhi et al., 2021). The aim of this argumentation strategy is to clarify and refine ideas, so that decisions can be made appropriately and well. One technique for assessing, grouping, and showing the quality or level of students' argumentation is to use the Toulmin's Argument model. The Toulmin model itself has been widely used by previous researchers in studying students' argumentation processes in mathematics (Amiruddin, et al., 2023; Tristanti, et al., 2016; Pramesti & Rosyidi, 2020).

The argumentation model proposed by Toulmin consists of 6 components, namely: (1) Claim, (2) Grounds/Data, (3) Warrant, (4) Backing, (5) Qualifier, and (6) Rebuttal. Each of these components has different functions and characteristics but supports each other (Amiruddin, et al., 2023; Conner et al., 2014; Metaxas, 2016; Sibel, 2018). A claim is a statement whose validity is being made, data is the foundation of an argument based on relevant facts to construct a claim. Statements that link data to claims are called warrants, statements that describe warrants as invalid are called rebuttals, while qualifications are statements that describe the certainty of a claim. Someone makes some kind of claim (could be an answer to a problem) so that their statement is considered part of the argument. Then the relationship between the data and the claim must be clarified through warrants about how the claim is made based on the data obtained. Warrants can take the form of formulas, definitions, axioms, or theorems or create analogies, pictures, or diagrams and graphs. In addition, there may be additional clarification, known as backing, about why the warrant is valid. A person's mathematical argumentation can be measured through questions that support students in providing claims, data and warrants. The schema consists of data, claims, warrants, support, rebuttal, and qualifications. (Tristanti, et al., 2016 ; Pramesti & Rosyidi, 2020) Toulmin's argumentation model is as shown in Figure 1 below :

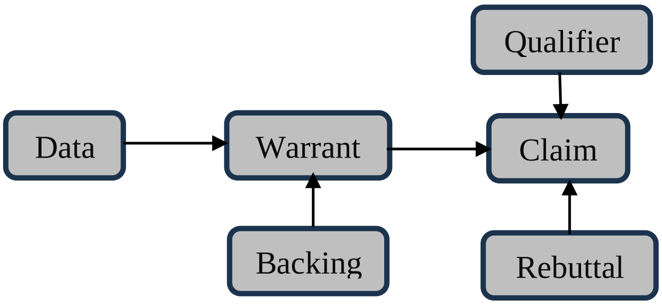


Figure 1. Toulmin's argumentation model

A person's ability to provide arguments cannot be separated from their ability to understand and solve the problems they face. Problem solving ability cannot be separated from a person's cognitive style (Akay, 1996; Bestiyana, 2018; Salam, 2020;). Cognitive style is a person's strategy in processing, cultivating, storing information from the environment that is used to solve problems (Astuti, 2019). The differences in students' cognitive styles cause differences in the level of their thinking abilities. Cognitive styles related to a person's habit of using their sensory organs are divided into two groups, namely visualizers and verbalizers (McEwan & Reynolds, 2007). Someone who has a visualizer cognitive style will tend to find it easier to receive, process, store and use information in the form of images or graphics. Meanwhile, someone who has a verbalizer cognitive style will tend to find it easier to receive, process, store and use information in the form of discussion in the form of text or writing. (McEwan & Reynolds, 2007; Salam, 2020).

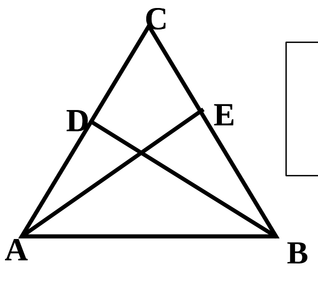
The results of the research show that the analogical reasoning of students with a visualizer cognitive style is superior to the analogical reasoning of students with a verbalizer cognitive style in solving geometric problems. Students with a visualizer cognitive style can achieve four components of analogical reasoning: encoding, inferring, mapping, and applying. Students with a verbalizer cognitive style only achieve two components, namely encoding and inferring (Shodikin, et al., 2023). Therefore, the author is interested in trying to describe the mathematical argumentation of students who have a visualizer cognitive style in proving the congruence of triangles. The author argues that the geometric proof related to the congruence of triangles is relevant to use as a test question for the title. With the reason that this question can explore the ability to carry out the process of formulating, applying theorems, and interpreting geometric images, where basic mathematical abilities such as reasoning and argumentation are needed in these three processes (Winer & Battista, 2022). A few misconceptions on class inclusion, especially when considering isosceles right triangles and obtuse triangles. Very few students correctly recognized the famous Pythagorean Theorem. Implications for more effective geometry teaching are considered

METHOD

This research is a qualitative descriptive study which aims to obtain an overview of the mathematical arguments of students who have a visualizer cognitive style in proving the congruence of triangles. Qualitative as a research method based on the philosophy of postpositivism, is used to research the conditions of natural objects, (as opposed to experiments) where the researcher is the key instrument, data collection techniques are carried out triangulation (combined), data analysis is inductive/qualitative, and Qualitative research results emphasize meaning rather than generalization (Kristina, 2020). The subjects of this research were students in the second semester of the 20223/2024 mathematics education study program at Madura University. The research was carried out by giving a written test in the form of a proof problem regarding the congruence of triangles. Subject selection was carried out by administering a visualiser verbaliser cognitive style determination test. to classify students based on cognitive style.

After obtaining the visualizer subjects, 3 research subjects were selected according to the conditions, 1 subject with high mathematical abilities (S1), 1 subject with moderate mathematical abilities (S2) and 1 subject with low mathematical abilities (S3) , and each subject had good communication skills. In this research, there are two types of instruments. The main instrument is the researcher himself because the researcher himself collects, analyzes and interprets the data. Meanwhile, the supporting instruments in this research are the Cognitive Style Classification Test (TGK), the test was developed by Mendelson, which is divided into 2 items, Items 1-10 (Verbaliser Items) and items 11-20 (Visualiser Items), Triangle Congruency Verification Test and Semi-structured Interview Guide. The research procedures carried out were as follows: a) Giving TGK to all students, b) Determining research subjects based on their level of mathematical ability and TKG, c) Subjects were asked to solve the problems given, d) Subjects were interviewed to reveal how they understood, processes and concepts used in solving the given problem.

The following are the test questions given:



Given $\overrightarrow{DA} \cong \overrightarrow{EB}$, $\overrightarrow{AE} \cong \overrightarrow{BD}$

Prove Δ ABC is an isosceles

Figure 2. triangle congruence proof test

RESULT

Mathematical Argumentation of Subject 1 (S1)

In explaining the completion of the written test on the questions given, S1 did not experience any difficulties. The stages of completing the proof using the two column proof system presented are quite systematic using axiomatic deductive principles. The premises given are in accordance with the statements in the question given and act as data that S1 uses to compile proof. The data that S1 uses appears in the written answers to S1's statements number 1 to number 3.

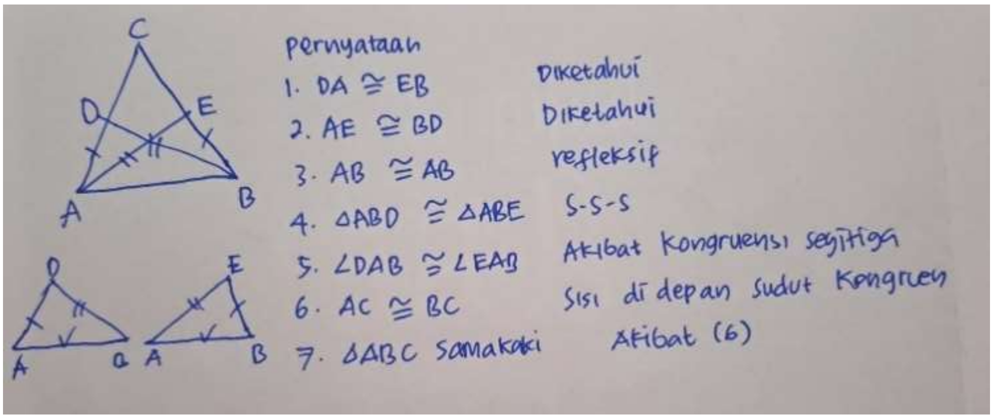


Figure 3. S1's test answers

S1 : "From the problem we are asked to prove that triangle ABC is isosceles, meaning we have to prove that side AC is congruent to BC"

From S1's statement above, it shows that the subject has planned a proof strategy which he adapts to the data he can use.

Data

S1 : " Because from the problem it is known that DA is congruent to EB and AE is congruent to BD, we can construct a proof that triangle ABD is congruent to triangle ABE by using the Side-Side-Side Theorem"

Claim Warrant

The subject then explained that to make it easier to compile the proof, S1 took out triangle ABD and triangle ABE from triangle ABC, as in Figure 3. The subject further explained that the reason for making these two triangles was so that the subject could easily determine the corresponding angles or sides of the two triangles. The subject's ability to reason and partition triangular images as above is related to the characteristics of the visualiser subject, where it is easier for the subject to solve problems by using images or partitions of a composed image. Subjects who have a visualizer cognitive style are more image-oriented and prefer visual games, such as putting together puzzles (Indahwati & Fetty, 2022; McEwan & Reynolds, 2007; Mendelson, 2004; Shodikin, et al. 2023)

From the 2 triangles that S1 took out from triangle ABC, S1 easily explained that with the concept of congruence based on the side-side-side theorem, where the three pairs of corresponding sides between triangle ABD and triangle ABE are congruent. This can be seen in S1's answer to statement number 4 . The subject adds statement number 3 as additional data that the subject obtained from separating the two triangles, that side AB is equally contained by both triangles so that based on the reflection properties it can be concluded that AB is congruent to AB. From S1's statement it is reflected that the visualizer subject is able to provide answers with simple (short) but clear explanations in solving geometric problems, and is able to build basic geometric skills (Winarso & Dewi, 2017)

In statement number 5, S1 easily concluded that the consequence of triangle ABE and triangle ABD being congruent is that angle DAB and angle EBA are congruent, so S1 quickly concluded that it was clear that triangle ABC was an isosceles triangle.

S1 : "Triangles ABD and ABE are congruent so that the corresponding angles are also congruent, so angle DAB is congruent to EAB, so side AC is also congruent to side BC" (Backing)

R : "Will there be any denial of this statement?

S1 : "None, because it is clear that angle A and angle B are congruent to isosceles triangle ABC, because side AC is in front of angle ABC and side BC is in front of angle A (no rebuttal)

Mathematical Argumentation of Subject 2 (S2)

In fact, the systematic proof that S2 subjects use is not much different from S2 subjects, but there are several stages that are not quite right. The data that S2 used in compiling the evidence in this proof is valid, in accordance with the statement given in the question. The statement that the subject gave in number 4 still cannot be used as a claim, because S2 should have given statement number 5 first as a warrant for statement number 4

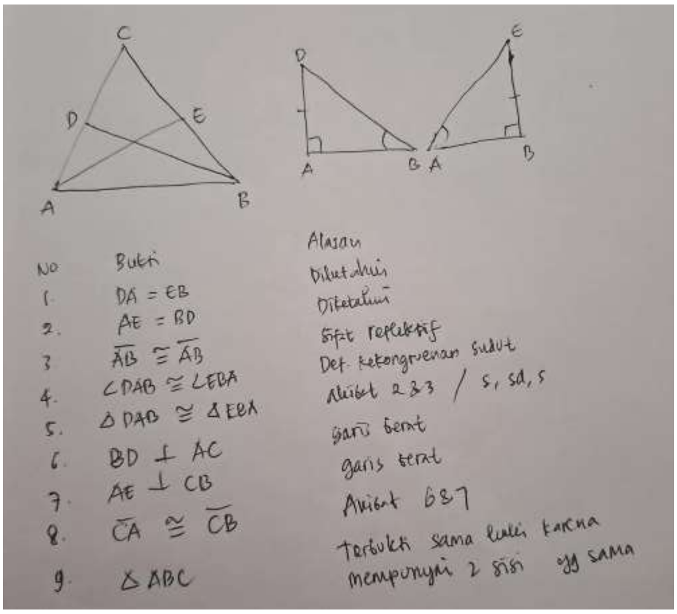


Figure 4. S2's test answers

R : "How can S2 draw the conclusion that angle DAB is congruent to angle EBA?"

S2 : "Because the two triangles I described are congruent, Ma'am" (triangle ABD and triangle ABE)

Then the subject explains using the side-side-side theorem that triangle ABD and triangle ABE are congruent. The subject should explain the statement first before drawing conclusions on statement number 4. So statement number 4 can be valid as a claim but statement number 5 must be included first.

Statements number 6 and 7 are invalid because the reasons or warrants and backing given are not based on facts or data contained in the test questions but are based on the subject's assumptions which are not based on reasons that match the data.

S2 : "BD is perpendicular to AC and AE is perpendicular to BC, as seen in the picture in the question"

R : "From which statement can you draw such a conclusion?"

S2 : "Just from the picture, ma'am, but I'm not sure and there's no certainty about perpendicularity"

S2's statement about this lack of confidence serves as a refutation of his argument, because the classification of "slip" is "characterized by the fact that the subject can correct himself. This is because students lack communication skills in expressing ideas or s2 knows the names of various definitions, theorems and postulates but does not know what they mean. (Casanova, et all., 2023). The subject was able to explain the concept of perpendicularity, but from the picture given in the question the subject was unsure about whether BD was perpendicular to AC because there was no data in the question that supported the subject's statement.

From the results of the interview with S2, it can be seen that the subject made an error in interpreting the image given in the question. This is related to the subject's mathematical representation ability. Mathematical representation abilities such as visual representation in the form of geometric patterns, representation of mathematical expressions in the form of geometric symbols such as points, lines, parallel lines, lines of equal length, angles, equal angles, congruence and triangles, as well as verbal representation in the form of text that explains problem solving stages. Factors that influence mathematical representation abilities include, lack of basic understanding and knowledge related to the context of geometric material, subjects are less able to present problems in geometric patterns, because subjects do not understand the problem, subjects use inaccurate mathematical representations, subjects are unable to manipulating geometric patterns to assist in solving problems (Rezki & Nurfadilah, 2018).

Statements number 6 and 7 are invalid (errors in interpreting geometric images). This is an example of a reasoning gap. S2's intuitive argumentation suggests that subjects understand which claims are valid, but have difficulty citing the correct justification from their axiomatic system of written evidence (Winer & Batista, 2022).

For statement number 8 subject, it cannot be used as a claim because it is based on invalid data numbers 6 and 7. It is possible that statement number 8 HR is his own assumption from the results of his reasoning due to an error in reading the data in the image provided by the question, so that his argument is invalid. Some of the students' symbolic representation errors in plane analytical geometry questions include not understanding the concept, errors in reading the data contained in the image, and errors in carrying out mathematical operations (Ikashaum et al., 2021), conclusion number 9 is also invalid.

Mathematical Argumentation of S3

In providing an argumentative explanation of the proof strategy, subject YP experienced several obstacles, namely related to the representation of images in the problem and the reasoning that the subject used. The subject's error in interpreting the data provided in the image renders the claim provided invalid. There is additional data that the subject provides from the results of the subject's assumptions in representing the image in the problem based on the subject's reasoning, but it is not accompanied by strong arguments. The subject was less careful in interpreting the picture and was fooled by the picture given in the question. YP experienced an error in representing the image in the question. YP considers point O to be the midpoint of sides AE and BD

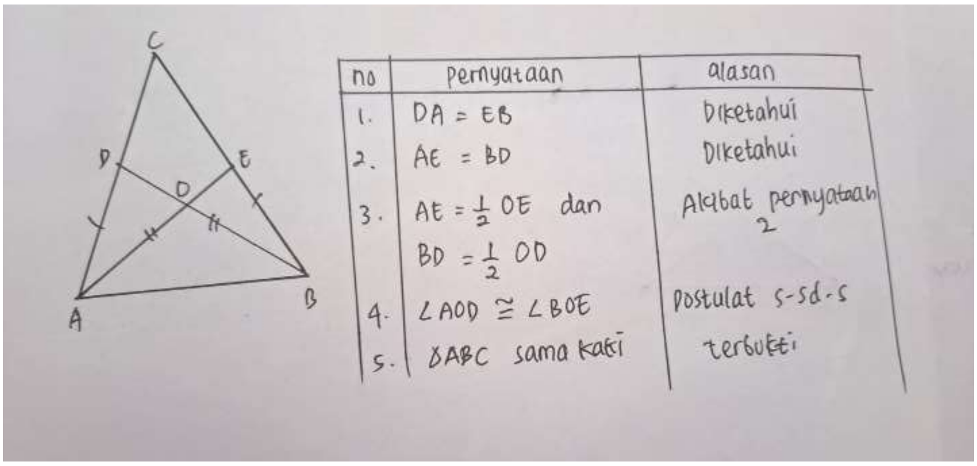


Figure 5. S3's test answers

R : "How can you conclude statement number 3?"
YP : "Because AE is congruent to BD and intersects at point O"

Then YP further explained that point O is the midpoint of AE and BD. "Because point O divides 2 sides of AC, so AE is equal to half of OE". The claim given by S3is clearly invalid, from the statement that O is the midpoint of AE and BD. From the answer S3 gave, it appears that the subject had difficulty explaining what he was thinking. The subject does not realize that what he explains does not match the statement the subject made in the column the subject wrote. Visualizer students are less able to assess, deny and give reasons. At the creation stage, it is considered sufficient, visualizer students are quite capable of designing solutions. However, they have difficulty visualizing from verbal form to visual form (Novitasari, Ria, & Sudiana, 2021).

Subject S3 claims that point O is the intersection point which is the midpoint of sides AE and BD, but the verbal explanation in this case is valid as a warrant is invalid. There is no data that can be used to support the claims made by the subject. The verbal explanation that the subject gave did not match what the subject wrote on the answer sheet. This is in line with research findings (Winer & Batista, 2022) which found that many students' reasoning conveyed in planning/oral explanations was not included in written evidence, resulting in fatal formalization and gaps in logical reasoning. Furthermore, the subject had difficulty understanding how to name angles, triangles, congruently.

DISCUSSION

From the presentation of the research results above, there are several interesting things to discuss. Subjects with a visualizer cognitive style with moderate mathematical abilities (S2) and those with low mathematical abilities (S3) apparently both experienced errors in representing geometric objects visually but at different points. This error results in the claim being made being invalid because the warrant given is incorrect. S2 and S3 subjects provide assumptions or interpret the images provided without validly collaborating with the data. Students' ability to explain geometric problems not only requires mastery of concepts, but also requires skills in representing problems into mathematical models (Indahwati & Fetty, 2022). Students with a visual learning style tend to be able to convey their ideas with visual representations and create pictures to facilitate solving problems. Students with a visual learning style have better visual representation abilities (Mafirah, et al., 2020). So there is a need for a strategy to overcome problems in visualizer subjects who experience problems in representing geometric images.

Mathematical representation here is a person's ability to re-express notations, symbols, tables, pictures, graphs, diagrams, equations or other mathematical expressions in different forms (Setyawati, 2020; Lutfi & Dasari, 2023). Therefore, this mathematical representation is very important to help students solve mathematical problems, including interpreting images of geometric objects (Clara & Dadan, 2023; Lette & Manoy, 2019; Lutfi & Khusna, 2021; Rohman , et al., 2018; Utami, et al., 2018). The results of research by several experts show that students' mathematical representation abilities are still in the very low category, especially for visual representation abilities (Clara & Dadan, 2023; Handayani, 2019). Furthermore, Mulyaningsih (2020) stated that pictorial representation is the mathematical representation ability that is least mastered by students. So it is still a task for teachers to hone students' mathematical representation skills.

There are several factors that influence students' ability to provide mathematical representations, including the subject being less able to present problems in geometric patterns, because the subject does not understand the problem (Rezki & Nurfadilah, 2018). One factor that can influence mathematical representation abilities is the characteristics of students' own way of thinking (Porter & Abdurrahman 2015). So, it is important to understand the unique characteristics of students' way of thinking so that teachers can increase students' understanding of mathematical concepts (Clara & Dadan, 2023). Therefore, it is necessary to design a learning model that is able to bridge each student with unique characteristics in their way of thinking so that learning objectives are achieved.

S1 is able to provide valid arguments accompanied by appropriate representation skills and of course the verbal explanations given are also correct. The ability to provide verbal explanations is related to the ability to understand and convey mathematical language. This mathematical language is related to vocabulary, symbols, and how to formulate an argument so that it can convey information correctly. So if this ability is not mastered, it will affect a person's ability to solve mathematical problems (Ernawati, et al., 2021). The ability to process words to convey a purpose or information is what is meant by literacy skills (Indahwati, et al., 2024). Many studies on analyzing students' literacy abilities have concluded that students' mathematical literacy abilities in solving mathematical problems are still relatively low (Astuti & Sabon, 2020; Chasanah, et al., 2024; Oktaviyanthi & Agus, 2019).

S2 and S3 experienced difficulty in conveying warrants for the claims they made because they had difficulty in expressing the reasons why the subjects provided additional data as a result of their inaccurate representation.

So in this case students' argumentation abilities are also influenced by their literacy abilities. Improving students' mathematical literacy skills needs to be a special concern for all of us, where each student has their own unique level of intelligence. It is necessary to plan learning models and techniques that can stimulate students' ability to dare to convey their ideas in every learning activity so that students' literacy skills can be honed and thus their argumentative abilities can also increase.

CONCLUSION

Based on the results and discussion above, the following are the mathematical argumentation profiles of the three research subjects. Subjects with high mathematical abilities provide proof based on correct claims, providing fairly complete data both from the question statement and additional data resulting from the subject's reasoning which is used to decide on the claim. The subject uses several logical statements to relate the data to the claim. The warrants are in the form of axioms and theorems as well as properties of triangle congruence. Next, provide backing by using conformity relationships in congruent triangles. For subjects with moderate ability and low ability to provide final answers or incorrect claims, they have incomplete data as a basis for deciding claims. This is because the subject is mistaken in representing the image in the question. For subjects with moderate mathematical abilities, the initial data is correct, but the additional data provided does not have any backing to support the warrant and even a rebuttal of the subject's own statement appears. Meanwhile, subjects with low mathematical abilities, apart from having difficulty in representing images in questions, also have deep mathematical literacy related to errors in explaining the visualizations in images.

From the explanation above, we can conclude that there are two aspects that need to be sharpened to improve students' mathematical argumentation abilities with the cognitive visualizer style in proving the congruence of triangles, namely the ability to represent and mathematical literacy. Teachers or lecturers must prepare learning models accompanied by problem solving questions that support students to express their reasoning accompanied by valid arguments by honing students' mathematical representation and literacy skills while still increasing students' understanding, increasing information retention, and developing cognitive skills. This can be done by increasing student involvement, high learning motivation, and assistance for students with a visual learning style. If there are students who have difficulty understanding visual learning material, alternative methods can be used or more interactive methods can be implemented. However, keep in mind that not all learning topics are suitable for visual learning methods. In any case, it is important to continually develop and adapt learning methods to ensure a more effective and rewarding learning experience for students. So by increasing the ability to visualize Fand literacy, it is hoped that students' argumentation abilities will also increase.

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